

HEAT TRANSFER FROM A TUBE BUNDLE IN
LONGITUDINALLY FLOWING WATER AND
ETHYLENE GLYCOL

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UDC 536.242

Heat transfer in tube bundles may be calculated from various formulas [1-6, 8-11], but there are no standard recommendations derived from analysis of all the published evidence. A survey has been made [1] of 14 papers for 21 tube assemblies carrying air and superheated steam with $Pr = 0.7-1$; here a survey is made of 10 further papers from the USA, USSR, France, and Czechoslovakia for 14 geometries with water flowing at Pr values of 1.1-6.5, and also for a mixture of 60% ethylene glycol and 40% water at a Pr of 11-18. The relative Nusselt number is here obtained as a function of the geometrical criterion in curvilinear form, as in [1]. The results from 24 sets of measurements by Soviet and other workers for 35 tube assemblies have given new and reliable recommendations for the heat calculation:

$$Nu_b = Nu_T \psi_T \psi_S,$$

which applies for $Re = 6 \cdot 10^3 - 10^6$; $Pr = 0.7-18$; $s_1 s_2 / d^2 = 1.2-6$; Nu_b is the Nusselt number for the bundle, while Nu_T is the Nusselt number for circular tubes from [12], or from the standard method of [7, 8], or from the standard method of [3]; ψ_T is a correction factor to the temperature factor in accordance with [6-8, 12], and ψ_S is a correction factor to the geometrical factor derived as in [1].

The method gives the best correction for the heat-transfer coefficient as a function of bundle geometry and also of the temperature factor, and the spread of most of the observed points around the line is only $\pm 4-6\%$.

These results show that it is unjustified to use formulas for tubes without correction factors at high s/d in order to calculate the mean heat transfer.

In practice, it appears best to perform such a generalization by assigning the physical properties of the liquid to the mean temperature and taking the hydraulic diameter of the bundle as the definitive dimension.

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THERMAL-RESISTANCE DETERMINATION WITH HEAT-SENSITIVE PAINTS

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The external thermal resistance of a component R_{t_0} is of interest in heat-exchanger design, since this can be reduced in order to minimize the temperature difference.

The usual methods of measuring R_{t_0} by means of contact transducers require special measuring instruments and are unsuitable for miniature electronic devices.

A method is given for measuring R_{t_0} by means of fusible temperature-indicating coatings.

Results are given from an experiment that confirms R_{t_0} can be determined in this way with an accuracy sufficient for most practical purposes.

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SURFACE TEMPERATURE OF STEEL DURING AIR-BLAST EROSION

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UDC 669.14.018

A fast gas stream containing an abrasive erodes the surface of stainless steel, which produces a temperature distribution in the contact zone, which must influence the processes in the wear zone, which are also influenced by the properties of the colliding materials and the failure mechanism.

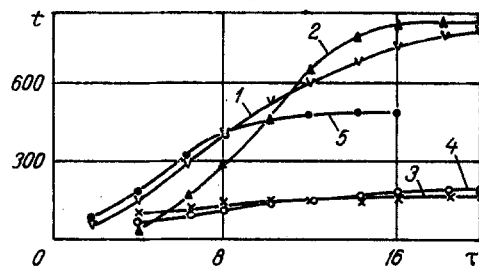


Fig. 1. Surface temperature of 1Kh13L and 1Kh18N9TL steels during abrasive blasting of $P_n = 6 \text{ kgf/cm}^2$, $\alpha = 60^\circ$, and K_b ($\text{kg} \cdot \text{cm}^{-2} \cdot \text{h}^{-1}$) of: 1, 2) 61; 3, 4) 5.1; 5) 144; 1, 3) open-circuit thermocouple at surface; 2, 4) thermocouple at depth of 1.0 mm; 5) thermocouple at depth of 2.0 mm.

A study has been made of the effects on temperature distribution at the surface and in lower layers for 1Kh13L and 1Kh18N9TL steels in jets of air containing abrasives at various angles of attack and abrasive concentrations; the temperatures were measured with thermocouples inserted to depths of 1-2 mm from the surface, and also with open-loop thermocouples, whose hot junctions were formed during wear at the surface; the temperature at the surface is determined by the angle of attack, the abrasive concentration, and the properties of the metal. Values up to 800°C can occur (Fig. 1).

These values for the temperature are to be considered as averages and below the actual values in microscopic volumes at the instant of collision with abrasive particles, since the heat released in wear is transmitted to the surrounding medium and absorbed by the eroded material. As the surface is vigorously cooled by the air, and the largest plastic deformation (and, consequently, the maximum heat release) occurs at a certain depth, particularly with work-hardening steels, one naturally finds the maximum mean temperature near the surface when the steady state is reached.

The temperatures found for 1Kh13L steel under identical wear conditions are higher than those for 1Kh18N9TL steel, which is due to the difference in properties.

NOTATION

α is the angle of attack at surface (deg);
 P_n is the air pressure (kgf/cm²) before nozzle;
 K_b is the abrasive concentration in blast (kg · cm⁻² · h⁻¹);
 t is the temperature (°C);
 τ is the time (sec).

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NONSTEADY FLOWS OF CONDENSING VAPOR IN LAVAL NOZZLES WITH COUNTERPRESSURE

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The nonsteady flow of condensing water vapor in Laval nozzles with an abrupt counterpressure within the channel (group III of the modes of nozzle operation of [1]) is analyzed in the article.

The studies were performed on a vapor-dynamic test stand whose working section was placed in the field of an IAB-51 shadow instrument [2]. An AEG movie camera was used for high-speed motion-picture photography of the nonsteady flow. The amplitude and frequency of the static pressure pulsations along the channel were measured with an LKh-610 piezoceramic pickup [3].

A supercritical supply of heat in the condensation zone leads to the periodic formation of transient shock waves in the region of small supersonic Mach numbers and, as a consequence, to pulsations of the stream parameters. This is the reason for the movement into the nozzle, with the same frequency, of a compression shock which forms upon the increase in counterpressure.

On the basis of an analysis of Topler motion pictures a difference was found in the wave structure of the stream in the following cases:

$$\left. \begin{array}{l} 1) \varepsilon_n > \varepsilon_a > \varepsilon_k \\ 2) \varepsilon_m > \varepsilon_a > \varepsilon_n \end{array} \right\} \begin{array}{l} P_0 < 1 \text{ bar,} \\ T_0 = T_s(P_0). \end{array}$$

Whereas in the first case the compression shock undergoes back-and-forth motion with slight variation in intensity, in the second case it periodically appears in a certain section of the channel, degenerating during its movement upstream. This occurs as a result of the fact that with a larger counterpressure ε_k the compression shock moves closer to the zone of spontaneous condensation and upon its interaction with the transient shock waves the expanding part of the nozzle immediately beyond the zone of spontaneous condensation ceases to operate in the diffusional mode.

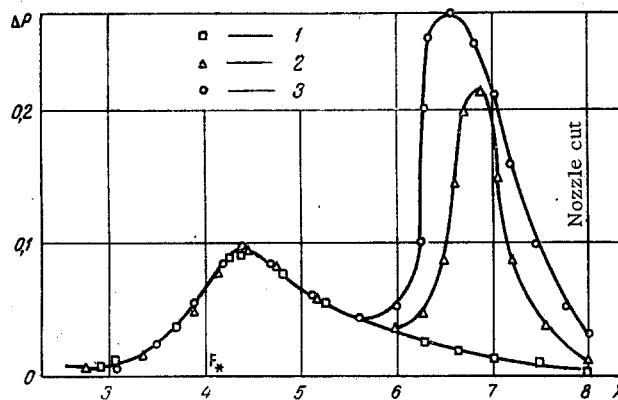


Fig. 1. Variation in relative amplitude of static pressure pulsations along the nozzle during nonsteady flow with condensation. Nozzle 3: 1) $\epsilon_a = 0.25 < \epsilon_k$; 2) $\epsilon_n > \epsilon_a = 0.596 > \epsilon_k$; 3) $\epsilon_m > \epsilon_a = 0.669 > \epsilon_n$; $P_0 = 0.9$ bar; $T_0 = T_{0S}$; $f = 830$ Hz; $y_* = 30$ mm.

A comparison of the results of the measurement of the amplitude of the static pressure pulsations (Fig. 1) for the cases of $\epsilon_a < \epsilon_k$ (curve 1) and $\epsilon_m > \epsilon_a > \epsilon_k$ (curves 2 and 3) shows that the presence of a compression shock leads to a considerable increase in the intensity of the pulsations within the nozzle.

NOTATION

$\epsilon_a = P_a/P_0$	is the dimensionless pressure beyond nozzle cut;
P_0, T_0	are the stagnation pressure and stagnation temperature at nozzle entrance, respectively;
ϵ_k	is the dimensionless pressure beyond nozzle cut at which a straight or curved shock is located in the exit cross section of the nozzle;
ϵ_n	is the limiting dimensionless pressure beyond nozzle cut which separates the two possible types of nonsteady flows of condensing vapor with counterpressure shocks within the channel;
ϵ_m	is the dimensionless limiting pressure beyond nozzle cut at which the counterpressure shock disappears;
f	is the frequency;
$\Delta P = \Delta p/P_0$	is the relative amplitude of static pressure pulsations;
$X = x/y_*$	is the relative longitudinal coordinate;
y_*	is the height of critical cross section of nozzle.

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EFFECT OF ELECTROSTATIC FIELD ON LIQUID
EVAPORATION FROM THE SURFACE OF
A CHARGED DROPLET

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Analytic investigations were carried out to determine the effect of an electrostatic field on the evaporation rate of a liquid from the surface of a single charged droplet. The motion of a charged droplet is considered in the backward flow of a heat carrier with electrostatic field forces taken into account. We obtained the equation of motion of the charged droplet for $Re < 200$

$$\frac{dv_d}{d\tau} = g - \frac{18\mu(v_d + v_c)}{\rho_e \delta_d^2} \pm \frac{0.82KE^2}{\delta_d \rho_l} \quad (1)$$

as well as of the velocity in the final form,

$$v_d = \frac{g\rho_l \delta_d^2 - 18\mu v_c \pm 0.82KE^2 \delta_d - \exp\left(\frac{18\mu\tau}{\rho_l \delta_d^2} - 18\mu C\right)}{18\mu}, \quad (2)$$

where

$$C = -\frac{1}{18\mu} \ln(g\rho_l \delta_d^2 - 18\mu v_c \pm 0.8 KE^2 \delta_d).$$

Using the electrostatic field forces an expression was given for the velocity:

$$N_{(E)} = 0.0745 \cdot \Delta P (v_{or} v)^{0.8}; \quad (3)$$

expressions are also given for the weight $W_{(E)}$ and for the share $\varphi_{(E)}$ of the evaporating liquid from the surface of a charged droplet:

$$W_{(E)} = \int_0^{\tau} N_{(E)} d\tau, \quad \varphi_{(E)} = \frac{W_{(E)}}{G}, \quad (4)$$

where

$$v_{or} = v_d + v_c$$

Analyzing Eqs. (1)-(4) it can be seen that the use of the methods of electron-ion techniques based on the use of electrostatic field forces in the heat- and mass-exchange treatment of dispersion media shows promise both from the point of view of making the processes more intensive and also as far as the reduction of the overall dimensions of the apparatus is concerned.

NOTATION

v_c is the heat-carrier velocity;
 E is the field intensity;
 g is the free-fall acceleration;
 δ_d is the droplet diameter;
 μ is the dynamic viscosity of heat carrier;
 ρ_l is the droplet density;
 G is the original droplet weight;
 ν is the specific weight of heat carrier;
 τ is the evaporation time.

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DISTRIBUTION OF AN IMPURITY IN A STREAM
WITH ALLOWANCE FOR SORPTION

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The equations for the transport of a substance in a stream and a sorbent have the following dimensionless form:

$$\frac{\partial c}{\partial \tau} + \frac{\partial c}{\partial \xi} + \frac{1}{m_0} \cdot \frac{\partial N}{\partial \tau} = \frac{\partial^2 c}{\partial \xi^2}, \quad c = c(\tau, \xi), \quad N = N(\tau, \xi), \quad (1)$$

$$\frac{\partial N}{\partial \tau} = \lambda(c - \alpha N), \quad \tau > 0, \quad \xi > 0.$$

We can differentiate the second equation of this system with respect to τ and introduce the new unknown function $v = \partial N / \partial \tau$ — the rate of change of the distribution of the substance in the sorbent. We have

$$\frac{\partial c}{\partial \tau} + \frac{\partial c}{\partial \xi} + \frac{1}{m_0} v = \frac{\partial^2 c}{\partial \xi^2}, \quad c = c(\tau, \xi), \quad v = v(\tau, \xi), \quad (2)$$

$$\frac{\partial v}{\partial \tau} = \lambda \left(\frac{\partial^2 c}{\partial \xi^2} - \frac{\partial c}{\partial \xi} - av \right), \quad a = \frac{1 + \alpha m_0}{m_0}.$$

For this system we consider the problem of the destruction of the initial equilibrium state, with a concentration c_0 of the substance in the stream and a concentration γc_0 of the substance in the sorbent, owing to the pumping through the cross section $\xi = 0$ of liquid with a concentration $c_* \neq c_0$ of the impurity. The corresponding limiting conditions have the form

$$\tau \leq 0, \quad 0 < \xi < \infty, \quad v = 0, \quad c = c_0, \quad (3)$$

$$\tau > 0, \quad \xi = 0, \quad c = c_*, \quad c(\tau, \infty) = c_0.$$

The solution of the problem (2) and (3) is sought in the form of series with respect to the parameter $\mu = 1/\lambda$, which is assumed to be small:

$$c(\tau, \xi) = \theta_0(\tau, \xi) + \mu \theta_1(\tau, \xi) + \dots + \mu^{q-1} \theta_{q-1}, \quad (4)$$

$$v(\tau, \xi) = v_0(\tau, \xi) + \mu v_1(\tau, \xi) + \dots + \mu^{q-1} v_{q-1}.$$

The first two terms of the expansion for $c(\tau, \xi)$ have the form

$$\theta_0(\tau, \xi) = c_0 + \frac{1}{2} (c_* - c_0) \left[\operatorname{erfc} \frac{1}{2} \left(\frac{\xi}{\sqrt{b\tau}} - \sqrt{b\tau} \right) + \exp \xi \operatorname{erfc} \frac{1}{2} \left(\frac{\xi}{\sqrt{b\tau}} + \sqrt{b\tau} \right) \right],$$

$$\theta_1(\tau, \xi) = c_* - \frac{(c_* - c_0) \xi}{4a^2 b^{3/2} m_0} \cdot \frac{1}{\sqrt{\pi \tau}} \left(\frac{1}{\tau} - \frac{\xi^2}{2b\tau} + \frac{b}{2} \right) \exp - \frac{(\xi - b\tau)^2}{4b\tau}, \quad (5)$$

$$b = 1 - \frac{1}{m_0 \mu}, \quad a = \frac{1 + \alpha m_0}{m_0}.$$

An estimate of the accuracy of the approximate solution found follows from the general theory of expansion in a series with respect to a small parameter with a leading derivative. The error of the approximation found for $c(\tau, \xi)$ with $\tau \in [0, \tau_*)$ and $0 < \mu < \mu_0$ is a quantity on the order of μ^2 .

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SIMULATION OF SOLIDIFICATION IN CONTINUOUS CASTING

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A mathematical model is presented for the solidification and cooling of a continuous steel casting of rectangular cross section crystallizing in the liquidus–solidus range; the model has been run on a computer

to predict the temperature distribution, the heat-flux density at the surface, the thickness of the crust at the exit from the crystallizer, and the depth of complete solidification at casting rates not yet attained.

The following is the differential equation for thermal conduction for the two-dimensional case subject to the above assumptions:

$$\rho c_e \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x} \left(\lambda_e \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_e \frac{\partial t}{\partial y} \right),$$

where

$$c_e = \begin{cases} c_s & \text{for } t < t_{so} \\ \frac{c_s + c_l}{2} + \frac{L}{t_{li} - t_{so}} & \text{for } t_{so} < t < t_{li} \\ c_l & \text{for } t > t_{li} \end{cases} \quad \lambda_e = \begin{cases} \lambda_s & \text{for } t < t_s \\ \lambda_s \psi + \lambda_l (1 - \psi) & \text{for } t_s < t < t_{li} \\ \lambda_l & \text{for } t > t_{li} \end{cases}$$

$$0 < x < l_w \quad 0 < n < l_n$$

The release of the latent heat of crystallization is incorporated by increasing the specific heat in the two-phase region by $L/(t_{li} - t_{so})$, while the effective thermal conductivity is dependent on the proportion ψ of solid phase in an elementary volume in the two-phase region.

There are four regions along the axis of the casting where the boundary conditions have to be specified separately;

- a) the zone of relatively close contact in the crystallizer (between the inner walls of the latter and the surface of the casting);
- b) the zone with a gas space in the lower part of the crystallizer; where shrinkage of the metal causes the casting to leave the wall;
- c) the zone of pumped water cooling of the broad faces;
- d) the air-cooled surface zone (radiation and convection cooling also), namely, the surfaces of the narrow faces and of the wide parts not flushed by the water.

The boundary conditions are symmetrical with respect to the geometrical axes of symmetry in the rectangular cross section, i.e., the solution is realized for one quarter of the cross section.

The model differs from standard ones in the literature in that one can determine the length of the zones of relatively close contact on the wide and narrow faces in the crystallizer on the basis of measurements on heat transfer to the cooling water for a particular machine.

Finite-difference numerical methods have been employed with an M-222 computer.

It is shown that the model is adequate to describe the actual solidification and cooling of a 0.24×1.71 m² casting, since the calculated values for the complete solidification depths and surface temperatures agree with the actual values. Results are given on the effects of casting rate on the complete-solidification depth.

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HEATING OR COOLING OF A LIQUID IN A CAVITY IN AN UNBOUNDED SOLID

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The temperature of a cooled (heated) liquid varies on account of three simultaneous processes: a) convective heat transfer between the liquid and the walls of the vessel; b) heating of the walls by thermal conduction (λ_T is the thermal conductivity of the wall); and c) temperature equalization throughout the volume V of the liquid by convection and thermal conduction.

A model for cooling is considered in which process c) occurs throughout the volume very much more rapidly than the temperature change in the liquid as a whole on account of heat lost through the wall. Then one can take the temperature $t(\tau)$ of the liquid as the same throughout the volume, apart from a thin layer near the wall, the temperature thus varying only with time. Also, the situation can be taken as one-dimensional.

Process b) is described by a known solution, which can be found, for instance, in Lykov's "Theory of Thermal Conduction" (Chap. 7, Sec. 7). This solution is used to write the equation for heat transfer as

$$-\frac{1}{m} \cdot \frac{dt}{d\tau} = t(\tau) - \int_0^{\tau} t(\tau - z) \varphi(z) dz, \quad (1)$$

where $\varphi(z)$ is a known function proportional to λ_T^{-1} and m is the cooling rate in the regular mode (for $\lambda_T = \infty$).

Equation (1) has been solved by Laplace transformation; the general solution is expressed in terms of the generalized variables as follows:

$$\theta = t(\tau)/t(0); \quad Bi^* = \kappa Bi; \quad Fo^* = \kappa^{-2} Fo,$$

with $t(\tau)$ reckoned from the wall temperature at time $\tau = 0$, while κ is the ratio of the bulk specific heats of the liquid and solid, the characteristic length in the problem being defined as $l = V/4F$, where F is the total surface of the cavity.

The solution takes the form

$$\theta = \sum_{i=1,2} A_i e^{\mu_i^2 Fo^*} \operatorname{erfc}(\mu_i \sqrt{Fo^*}) \quad \text{for } Bi^* \neq 1, \quad (2a)$$

$$\theta = (1 - Fo^*/2) \exp\left(-\frac{Fo^*}{4}\right) \operatorname{erfc}\left(\frac{Fo^*}{4}\right)^{1/2} + (Fo^*/\pi)^{1/2} \quad \text{for } Bi^* = 1, \quad (2b)$$

where $A_{1,2}(Bi^*)$ and $\mu_{1,2}(Bi^*)$ are functions of Bi^* of the form:

$$2A_{1,2}(x) = 1 \pm (1 - 1/x)^{-1/2}, \quad 2\mu_{1,2}(x) = x[1 \mp (1 - 1/x)^{1/2}].$$

Solutions (2a) and (2b) have been examined for various values of Bi^* ; in the case $Fo^* \gg 1$, simple asymptotic formulas may be derived from (2).

An example is given of the calculation of the temperature distribution in an underground ethylene store on the basis of (2a).

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SOLIDIFICATION OF AN INHOMOGENEOUS HALF-SPACE WITH STEFAN'S-LAW EMISSION FROM THE SURFACE

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UDC 836.24

An inhomogeneous half-space consists of a three-dimensional layer and a semiinfinite region, the two differing in thermal characteristics. Initially, the system is uniformly heated to a temperature T_0 , and the bodies are in contact. Thermal radiation in accordance with Stefan's law occurs from the surface $x = 0$ into a medium at zero temperature; it is assumed that the temperatures of the bodies are dependent on the time t and coordinate x , and then the problem is to integrate a system of differential equations subject to the nonlinear boundary condition

$$\left. \frac{\partial u_1}{\partial x} \right|_{x=0} = v(t) = c\sigma [u_1(0, t)]^4 \quad (1)$$

and the conjugation conditions at the boundary $x = 1$; a solution convenient for small t takes the form

$$u_1(x, t) = T_0 - \frac{a_1}{\sqrt{\pi}} \int_0^t \frac{v(\tau)}{\sqrt{t-\tau}} \sum_{n=1}^{\infty} \left(h^n e^{-\frac{(2n t - x)^2}{4a_1^2(t-\tau)}} + h^{n-1} e^{-\frac{[2(n-1)t+x]^2}{4a_1^2(t-\tau)}} \right) d\tau. \quad (2)$$

If t is large it is convenient to use the solution

$$u_1(x, t) = T_0 - \frac{2a_1^2}{l} \int_0^t v(\tau) \left(1 + 2 \sum_{n=1}^{\infty} h^n e^{-\frac{a_1^2 n^2 \tau^2 (t-\tau)}{l^2}} \right) d\tau, \quad (3)$$

where $h = \frac{\frac{k_1 a_2}{k_2 a_1} - 1}{\frac{k_1 a_2}{k_2 a_1} + 1}$.

Tikhonov's method may be used to reduce the problem to one of successive approximation for a non-linear Volterra integral equation.

Numerical values are presented together with the trend in the successive approximations.

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